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Towards Precise Determinations of the CKM Matrix without Hadronic Uncertainties*

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Abstract

We illustrate how the measurements of the CP asymmetries in $B^0_{d,s}$-decays together with a measurement of $Br(K_L \to \pi^\circ\nu\bar{\nu})$ or $Br(K^+ \to \pi^+\nu\bar{\nu})$ and the known value of $\mid V_{us} \mid$ can determine all elements of the Cabibbo-Kobayashi-Maskawa matrix essentially without any hadronic uncertainties. An analysis using the ratio x_d/x_s of $B_d - \bar{B}_d$ to $B_s - \bar{B}_s$ mixings is also presented.

1. Setting the Scene

An important target of particle physics is the determination of the unitary 3×3 Cabibbo-Kobayashi-Maskawa matrix which parametrizes the charged current interactions of quarks:

$$J^{cc}_\mu = (\bar{u}, \bar{c}, \bar{t})_L \gamma_\mu \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L \qquad (1)$$

It is customery these days to parametrize these matrix by the four Wolfenstein parameters $(\lambda, A, \varrho, \eta)$. In particular one has

$$\mid V_{us} \mid = \lambda \qquad \mid V_{cb} \mid = A\lambda^2 \qquad (2)$$

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and

$$V_{ub} = A\lambda^3(\varrho - i\eta) \qquad V_{td} = A\lambda^3(1 - \bar{\varrho} - i\bar{\eta}) \qquad (3)$$

Here following [1] we have introduced

$$\bar{\varrho} = \varrho(1 - \frac{\lambda^2}{2}) \qquad \bar{\eta} = \eta(1 - \frac{\lambda^2}{2}). \qquad (4)$$

which allows to improve the accuracy of the Wolfenstein parametrization.

From tree level K decays sensitive to V_{us} and tree level B decays sensitive to V_{cb} and V_{ub} we have:

$$\lambda = 0.2205 \pm 0.0018 \qquad \mid V_{cb} \mid = 0.039 \pm 0.004 \qquad (5)$$

$$R_b \equiv \sqrt{\bar{\varrho}^2 + \bar{\eta}^2} = (1 - \frac{\lambda^2}{2}) \frac{1}{\lambda} \left| \frac{V_{ub}}{V_{cb}} \right| = 0.36 \pm 0.14 \qquad (6)$$

corresponding to

$$\left| \frac{V_{ub}}{V_{cb}} \right| = 0.08 \pm 0.03 \qquad (7)$$

R_b is just the length of one side of the rescaled unitarity triangle in which the length of the side on the $\bar{\varrho}$ axis is equal unity. The length of the third side is governed by $\mid V_{td} \mid$ and is given by

$$R_t \equiv \sqrt{(1-\bar{\varrho})^2 + \bar{\eta}^2} = \frac{1}{\lambda} \left| \frac{V_{td}}{V_{cb}} \right| \tag{8}$$

In order to find R_t one has to go beyond tree level decays.

As we have seen at this conference a large part in the errors quoted in (5), (6) and (7) results from theoretical (hadronic) uncertainties. Consequently even if the data from CLEO II improves in the future, it is difficult to imagine at present that in the tree level B-decays a better accuracy than $\Delta \mid V_{cb} \mid = \pm 2 \cdot 10^{-3}$ and $\Delta \mid V_{ub}/V_{cb} \mid = \pm 0.01$ ($\Delta R_b = \pm 0.04$) could be achieved unless some dramatic improvements in the theory will take place.

The question then arises whether it is possible at all to determine the CKM parameters without any hadronic uncertainties. The aim of this contribution is to demonstrate that this is indeed possible. To this end one has to go to the loop induced decays or transitions governed by short distance physics. We will see that in this manner clean and precise determinations of $\mid V_{cb} \mid$, $\mid V_{ub}/V_{cb} \mid$, $\mid V_{td} \mid$, ϱ and η can be achieved. Since the relevant measurements will take place only in the next decade, what follows is really a 21st century story.

It is known that many loop induced decays contain also hadronic uncertainties [2]. Examples are $B^0 - \bar{B}^0$ mixing, ε_K and ε'/ε. Let us in this connection recall the expectations from a "standard" analysis of the unitarity triangle which is based on ε_K, x_d giving the size of $B^0 - \bar{B}^0$ mixing, $\mid V_{cb} \mid$ and $\mid V_{ub}/V_{cb} \mid$ with the last two extracted from tree level decays. As a recent analysis [1] shows, even with optimistic assumptions about the theoretical and experimental errors it will be difficult to achieve the accuracy better than $\Delta\varrho = \pm 0.15$ and $\Delta\eta = \pm 0.05$ this way. Therefore in what follows we will only discuss the four finalists in the field of weak decays which essentially are free of hadronic uncertainties.

2. Finalists

2.1. CP-Asymmetries in B^o-Decays

The CP-asymmetry in the decay $B_d^\circ \to \psi K_S$ allows in the standard model a direct measurement of the angle β in the unitarity triangle without any theoretical uncertainties [3]. Similarly the decay $B_d^\circ \to \pi^+\pi^-$ gives the angle α, although in this case strategies involving other channels are necessary in order to remove hadronic uncertainties related to penguin contributions [4]. The determination of the angle γ from CP asymmetries in neutral B-decays is more difficult but not impossible [5]. Also charged B decays could be useful in this respect [6]. We have for instance

$$A_{CP}(\psi K_S) = -\sin(2\beta)\frac{x_d}{1+x_d^2}, \tag{9}$$

$$A_{CP}(\pi^+\pi^-) = -\sin(2\alpha)\frac{x_d}{1+x_d^2} \tag{10}$$

where we have neglected QCD penguins in $A_{CP}(\pi^+\pi^-)$. Since in the usual unitarity triangle one side is known, it suffices to measure two angles to determine the triangle completely. This means that the measurements of $\sin 2\alpha$ and $\sin 2\beta$ can determine the parameters ϱ and η. The main virtues of this determination are as follows:

- No hadronic or $\Lambda_{\overline{MS}}$ uncertainties.
- No dependence on m_t and V_{cb} (or A).

2.2. $K_L \to \pi^o \nu\bar{\nu}$

$K_L \to \pi^o \nu\bar{\nu}$ is the theoretically cleanest decay in the field of rare K-decays. $K_L \to \pi^o \nu\bar{\nu}$ is dominated by short distance loop diagrams involving the top quark and proceeds almost entirely through direct CP violation. The last year calculations [7, 8] of next-to-leading QCD corrections to this decay considerably reduced the theoretical uncertainty due to the choice of the renormalization scales present in the leading order expression [9]. Typically the uncertainty in $Br(K_L \to \pi^0\nu\bar{\nu})$ of $\pm 10\%$ in the leading order is reduced to $\pm 1\%$. Since the relevant hadronic matrix elements of the weak current $\bar{s}\gamma_\mu(1-\gamma_5)d$ can be measured in the leading decay $K^+ \to \pi^0 e^+ \nu$, the resulting theoretical expression for Br($K_L \to \pi^o\nu\bar{\nu}$) is only a function of the CKM parameters, the QCD scale $\Lambda_{\overline{MS}}$ and m_t. The long distance contributions to $K_L \to \pi^o\nu\bar{\nu}$ are negligible. We have then:

$$Br(K_L \to \pi^0\nu\bar{\nu}) = 1.50 \cdot 10^{-5} \eta^2 \mid V_{cb} \mid^4 x_t^{1.15} \tag{11}$$

where $x_t = m_t^2/M_W^2$ with $m_t \equiv \bar{m}_t(m_t)$. The main features of this decay are:

- No hadronic uncertainties
- $\Lambda_{\overline{MS}}$ and renormalization scale uncertainties at most $\pm 1\%$.
- Strong dependence on m_t and V_{cb} (or A).

2.3. $K^+ \to \pi^+ \nu\bar{\nu}$

$K^+ \to \pi^+\nu\bar{\nu}$ is CP conserving and receives contributions from both internal top and charm exchanges. The last year calculations [7, 8, 10] of next-to-leading QCD corrections to this decay considerably reduced the theoretical uncertainty due to the choice of the renormalization scales present in the leading order expression [9]. Typically the uncertainty in $Br(K^+ \to \pi^+\nu\bar{\nu})$ of $\pm 20\%$

in the leading order is reduced to $\pm 5\%$. The long distance contributions to $K^+ \to \pi^+ \nu\bar{\nu}$ have been considered in [11] and found to be very small: two to three orders of magnitude smaller than the short distance contribution at the level of the branching ratio. $K^+ \to \pi^+ \nu\bar{\nu}$ is then the second best decay in the field of rare decays. Compared to $K_L \to \pi^0 \nu\bar{\nu}$ it receives additional uncertainties due to m_c and the related renormalization scale. Also its QCD scale dependence is stronger. Explicit expressions can be found in [10, 12]. The main features of this decay are:

- Hadronic uncertainties below 1%
- $\Lambda_{\overline{MS}}$, m_c and renormalization scales uncertainties at most $\pm(5-10)\%$.
- Strong dependence on m_t and V_{cb} (or A).

2.4. $B^o - \bar{B}^o$ Mixing

Measurement of $B_d^o - \bar{B}_d^o$ mixing parametrized by x_d together with $B_s^o - \bar{B}_s^o$ mixing parametrized by x_s allows to determine R_t:

$$R_t = \frac{1}{\sqrt{R_{ds}}} \sqrt{\frac{x_d}{x_s}} \frac{1}{\lambda} \tag{12}$$

with $R_{d,s}$ summarizing SU(3)–flavour breaking effects. Note that m_t and V_{cb} dependences have been eliminated this way and R_{ds} contains much smaller theoretical uncertainties than the hadronic matrix elements in x_d and x_s separately. Provided x_d/x_s has been accurately measured a determination of R_t within $\pm 10\%$ should be possible. The main features of x_d/x_s are:

- No $\Lambda_{\overline{MS}}$, m_t and V_{cb} dependence.
- Hadronic uncertainty in SU(3)–flavour breaking effects of roughly $\pm 10\%$.

Because of the last feature, x_d/x_s cannot fully compete in the clean determination of CKM parameters with CP asymmetries in B-decays and with $K_L \to \pi^0 \nu\bar{\nu}$. Although $K^+ \to \pi^+ \nu\bar{\nu}$ has smaller hadronic uncertainties than x_d/x_s, its dependence on $\Lambda_{\overline{MS}}$ and m_c puts it in the same class as x_d/x_s [2].

3. $\sin(2\beta)$ from $K \to \pi\nu\bar{\nu}$

It has been pointed out in [13] that measurements of $Br(K^+ \to \pi^+ \nu\bar{\nu})$ and $Br(K_L \to \pi^0 \nu\bar{\nu})$ could determine the unitarity triangle completely provided m_t and V_{cb} are known. In view of the strong dependence of these branching ratios on m_t and V_{cb} this determination is not precise however [12]. On the other hand it has been noticed recently [12] that the m_t and V_{cb} dependences drop out in the evaluation of $\sin(2\beta)$. Introducing the "reduced" branching ratios

$$B_+ = \frac{Br(K^+ \to \pi^+ \nu\bar{\nu})}{4.64 \cdot 10^{-11}} \qquad B_L = \frac{Br(K_L \to \pi^0 \nu\bar{\nu})}{1.94 \cdot 10^{-10}} \tag{13}$$

one finds

$$\sin(2\beta) = \frac{2 r_s(B_+, B_L)}{1 + r_s^2(B_+, B_L)} \tag{14}$$

where

$$r_s(B_+, B_L) = \frac{\sqrt{(B_+ - B_L)} - P_0(K^+)}{\sqrt{B_L}} \tag{15}$$

so that $\sin(2\beta)$ does not depend on m_t and V_{cb}. Here $P_0(K^+) = 0.40 \pm 0.09$ [10, 12] is a function of m_c and $\Lambda_{\overline{MS}}$ and includes the residual uncertainty due to the renormalization scale μ. Consequently $K^+ \to \pi^+ \nu\bar{\nu}$ and $K_L \to \pi^0 \nu\bar{\nu}$ offer a clean determination of $\sin(2\beta)$ which can be confronted with the one possible in $B^0 \to \psi K_S$ discussed above. Any difference in these two determinations would signal new physics. Choosing $Br(K^+ \to \pi^+ \nu\bar{\nu}) = (1.0 \pm 0.1) \cdot 10^{-10}$ and $Br(K_L \to \pi^0 \nu\bar{\nu}) = (2.5 \pm 0.25) \cdot 10^{-11}$, one finds [12]

$$\sin(2\beta) = 0.60 \pm 0.06 \pm 0.03 \pm 0.02 \tag{16}$$

where the first error is "experimental", the second represents the uncertainty in m_c and $\Lambda_{\overline{MS}}$ and the last is due to the residual renormalization scale uncertainties. This determination of $\sin(2\beta)$ is competitive with the one expected at the B-factories at the beginning of the next decade.

4. Precise Determinations of the CKM Matrix

Using the first two finalists and $\lambda = 0.2205 \pm 0.0018$ [14] it is possible to determine all the parameters of the CKM matrix without any hadronic uncertainties [15]. With $a \equiv \sin(2\alpha)$, $b \equiv \sin(2\beta)$ and $Br(K_L) \equiv Br(K_L \to \pi^0 \nu\bar{\nu})$ one determines ϱ, η and $|V_{cb}|$ as follows [15]:

$$\bar{\varrho} = 1 - \bar{\eta} r_+(b) \quad , \quad \bar{\eta} = \frac{r_-(a) + r_+(b)}{1 + r_+^2(b)} \tag{17}$$

$$|V_{cb}| = 0.039 \sqrt{\frac{0.39}{\eta}} \left[\frac{170\ GeV}{m_t}\right]^{0.575} \left[\frac{Br(K_L)}{3 \cdot 10^{-11}}\right]^{1/4} \tag{18}$$

where

$$r_\pm(z) = \frac{1}{z}(1 \pm \sqrt{1 - z^2}) \qquad z = a, b \tag{19}$$

We note that the weak dependence of $|V_{cb}|$ on $Br(K_L \to \pi^0 \nu\bar{\nu})$ allows to achieve high accuracy for this CKM element even when $Br(K_L \to \pi^0 \nu\bar{\nu})$ is not measured precisely.

As illustrative examples we consider in table 1 three scenarios. The first four rows give the assumed

	Central	I	II	III
$\sin(2\alpha)$	0.40	±0.08	±0.04	±0.02
$\sin(2\beta)$	0.70	±0.06	±0.02	±0.01
m_t	170	±5	±3	±3
$10^{11} Br(K_L)$	3	±0.30	±0.15	±0.15
ϱ	0.072	±0.040	±0.016	±0.008
η	0.389	±0.044	±0.016	±0.008
$\mid V_{ub}/V_{cb} \mid$	0.087	±0.010	±0.003	±0.002
$\mid V_{cb} \mid /10^{-3}$	39.2	±3.9	±1.7	±1.3
$\mid V_{td} \mid /10^{-3}$	8.7	±0.9	±0.4	±0.3
$\mid V_{cb} \mid /10^{-3}$	41.2	±4.3	±3.0	±2.8
$\mid V_{td} \mid /10^{-3}$	9.1	±0.9	±0.6	±0.6

Table 1. Determinations of various parameters in scenarios I-III

	Central	I	II	III
R_t	1.00	±0.10	±0.05	±0.03
$\sin(2\beta)$	0.70	±0.06	±0.02	±0.01
m_t	170	±5	±3	±3
$10^{11} Br(K_L)$	3	±0.30	±0.15	±0.15
ϱ	0.076	±0.111	±0.053	±0.031
η	0.388	±0.079	±0.033	±0.019
$\mid V_{ub}/V_{cb} \mid$	0.087	±0.014	±0.005	±0.003
$\mid V_{cb} \mid /10^{-3}$	39.3	±5.7	±2.6	±1.8
$\mid V_{td} \mid /10^{-3}$	8.7	±1.2	±0.6	±0.4
$\mid V_{cb} \mid /10^{-3}$	41.3	±5.8	±3.7	±3.3
$\mid V_{td} \mid /10^{-3}$	9.1	±1.3	±0.8	±0.7

Table 2. As in table 1 but with $\sin(2\alpha)$ replaced by R_t.

input parameters and their experimental errors. The remaining rows give the results for selected parameters. Further results can be found in [15]. The accuracy in the scenario I should be achieved at B-factories, HERA-B, at KAMI and at KEK. Scenarios II and III correspond to B-physics at Fermilab during the Main Injector era and at LHC respectively. At that time an improved measurement of $Br(K_L \to \pi^0 \nu\bar{\nu})$ should be aimed for. Table 1 shows very clearly the potential of CP asymmetries in B-decays and of $K_L \to \pi^0\nu\bar{\nu}$ in the determination of CKM parameters. It should be stressed that this high accuracy is not only achieved because of our assumptions about future experimental errors in the scenarios considered, but also because $\sin(2\alpha)$ is a very sensitive function of ϱ and η [1], $Br(K_L \to \pi^0\nu\bar{\nu})$ depends strongly on $\mid V_{cb} \mid$ and most importantly because of the clean character of the quantities considered.

It is instructive to investigate whether the use of $K^+ \to \pi^+\nu\bar{\nu}$ instead of $K_L \to \pi^0\nu\bar{\nu}$ would also give interesting results for V_{cb} and V_{td}. We again consider scenarios I-III with $Br(K^+ \to \pi^+\nu\bar{\nu}) = (1.0\pm0.1)\cdot 10^{-10}$ for the scenario I and $Br(K^+ \to \pi^+\nu\bar{\nu}) = (1.0 \pm 0.05) \cdot 10^{-10}$ for scenarios II and III in place of $Br(K_L \to \pi^0\nu\bar{\nu})$ with all other input parameters unchanged. An analytic formula for $\mid V_{cb} \mid$ can be found in [15]. The results for ϱ, η, and $\mid V_{ub}/V_{cb} \mid$ remain of course unchanged. In the last two rows of table 1 we show the results for $\mid V_{cb} \mid$ and $\mid V_{td} \mid$. We observe that due to the uncertainties present in the charm contribution to $K^+ \to \pi^+\nu\bar{\nu}$, which was absent in $K_L \to \pi^0\nu\bar{\nu}$, the determinations of $\mid V_{cb} \mid$ and $\mid V_{td} \mid$ are less accurate. If the uncertainties due to the charm mass and $\Lambda_{\overline{MS}}$ are removed one day this analysis will be improved [15].

An alternative strategy is to use the measured value of R_t instead of $\sin(2\alpha)$. Then (17) is replaced by

$$\bar{\varrho} = 1 - \bar{\eta} r_+(b) \quad , \quad \bar{\eta} = \frac{R_t}{\sqrt{2}} \sqrt{br_-(b)} \tag{20}$$

The result of this exercise is shown in table 2. We observe that even with rather optimistic assumptions on the accuracy of R_t, this determination of CKM parameters cannot fully compete with the previous one. Again the last two rows give the results when $K_L \to \pi^0\nu\bar{\nu}$ is replaced by $K^+ \to \pi^+\nu\bar{\nu}$.

5. Final Remarks

- Precise measurements of all CKM parameters without hadronic uncertainties are possible.
- Such measurements are essential for the tests of the standard model. Of particular interest will be the comparison of $\mid V_{cb} \mid$ determined as suggested here with the value of this CKM element extracted from tree level semi-leptonic B-decays. Since in contrast to $K_L \to \pi^0\nu\bar{\nu}$ and $K^+ \to \pi^+\nu\bar{\nu}$, the tree-level decays are to an excellent approximation insensitive to any new physics contributions from very high energy scales, the comparison of these two determinations of $\mid V_{cb} \mid$ would be a good test of the standard model and of a possible physics beyond it.

Precise determinations of all CKM parameters without hadronic uncertainties along the lines presented here can only be realized if the measurements of CP asymmetries in B-decays and the measurements of $Br(K_L \to \pi^0\nu\bar{\nu})$, $Br(K^+ \to \pi^+\nu\bar{\nu})$ and x_d/x_s can reach the desired accuracy. All efforts should be made to achieve this goal.

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